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OPTIMIZATION OF NODE FIXING IN WIRELESS SENSOR NETWORK USING CONNECTED DOMATIC NUMBER OF A GRAPH

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Abstract

Wireless Sensor Network (WSN) is composed of miniature sensor devices which include tiny sensor and small batteries with energy, computation and communication constraints. Care must be taken in placing the nodes for effective optimization of accessing the resources in the network. In this paper, merging of two arbitrary wireless sensor network is considered and aims at optimizing the placement of nodes by utilizing the concept of connected domatic number of a graph.

Keywords: Wireless Sensor Network, Dominating Set, Domatic Number, Connected Domatic set, Domatic Partition.

Introduction

Graph theory is one of the hottest research areas of modern mathematics which has seen a magnificent growth due to the number of applications in computer and communication, molecular physics and chemistry, social networks, biological sciences, computational linguistics, and in other numerous fields. In graph theory, one of the extensively researched branches is domination in graph. In graph theory, a set $S \subseteq V$ is said to be a dominating set, if every vertex in V-S is adjacent to at least one vertex in S. The minimum cardinality taken over all minimal dominating set is called the domination number of G and is denoted by $\gamma(G)$ [1]. A dominating set is called a connected dominating set if the subgraph $\langle S \rangle$ induced by S is connected. The connected domination number $\gamma_c(G)$ is the minimum number of vertices in a connected dominating set in graph G [2]. A domatic partition of a graph G=(V,E) is a partition of V into disjoint sets $V_1, V_2, V_3, \ldots V_k$ such that each V_j is a dominating set for G. The maximum number of dominating sets in which the vertex set of a graph G can be partitioned is called the domatic number of graph G, and it is denoted by dom(G) or d(G) [3].

The concept of domatic partitioning plays an important role in locating the resources in a network. Let us assume that a node in the network can access only the resources present in the neighboring nodes or itself. A network may contain several essential types of resources to be used. If a particular resource is needed to be accessed from every node, then, the dominating set of the network must possess the copy of that resource. This particular resource which is to be accessed must occupy the dominating set of the network. If each node has bounded capacity, the amount of resource to be occupied in a node is limited. If each node can hold only a single resource then the dominating set will support the maximum number of resources which is equal to the domatic number of the graph [4].

Characterization of Connected Domatic Number

We review some elementary facts about dominating sets and domatic partitions, in light of the novelty of the problem for many readers.

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Results on Dominating Set

Dominating sets satisfy a monotonicity property with regards to vertex additions: if D is a dominating set and D' ⊃ D, the D' is also a dominating set. This implies that if a graph contains k disjoint dominating sets, then its domatic number is at most k; those nodes not belonging to any of the k sets can be added arbitrarily to the sets to form a proper partition of the vertex set. The domatic number can be alternatively defined as the maximum number of disjoint dominating sets. Every graph G satisfies $D(G) \geq 1$, and unless G contains an isolated node, $D(G) \geq 2$. On the other hand, $D(G) \leq \delta + 1$, where δ is the minim degree; the reason being that a node of minimum degree must have some neighbor in each of the disjoint dominating sets [5].

Results on Domatic Number

In wireless sensor networks, rotating dominating sets periodically is an important technique, for balancing energy consumption of nodes and hence maximizing the lifetime of the networks. This technique can be abstracted as the domatic partition problem, which partitions the set of nodes in networks into disjoint dominating sets. Through rotating each dominating set in the dramatic partition periodically, the energy consumption of the nodes can be greatly balanced and the lifetime of the network can be prolonged for setting up sleep scheduling in sensor networks [6]. To formulate the problem as an instance of the fractional domatic partition problem and obtain a distributed approximation algorithm, by applying linear programming approximation techniques [7].

Wireless ad hoc and sensor networks (WSNs) often require a connected dominating set (CDS) as the underlying virtual backbone for efficient routing [8]. Nodes in a CDS have extra computation and communication load for their role as dominator, subjecting them to an early exhaustion of their battery. A simple mechanism to address this problem is to switch from one CDS to another fresh CDS, rotating the active CDS through a disjoint set of CDSs. This gives rise to the connected domatic partition problem, which essentially involves partitioning the nodes V(G) of a graph G into node disjoint CDSs [9] [19].

A set of vertices in a graph is a dominating set if every vertex outside the set has a neighbor in the set. The domatic number problem is that of partitioning the vertices of a graph into the maximum number of disjoint dominating sets. Let n denote the number of vertices, δ the minimum degree, and Δ the maximum degree [10] [19].

Every graph has a domatic partition with $(1 - o(1))(\delta + 1) / \ln n$ dominating sets, and moreover, that such a domatic partition can be found in polynomial time. This implies a $(1 + o(1)) \ln n$ approximation algorithm for a domatic number, since the domatic number is always at most $\delta + 1$. This approximation is applicable for set cover by combining multi-prover protocols with aero-knowledge techniques [11] [19].

Definitions

In this paper, we are considering simple connected and undirected graphs. Applying a Cartesian product on the complete graph, and finding the connected domatic number in the resulting graph. Throughout this paper, we consider a graph a two graph $G=(p_1,q_1)$ and $H=(p_2,q_2)$ where $p_2 \leq p_1$ for applying a Cartesian product on G and H which we consider as G (H= (p, q) where $p=p_1\times p_2$ is the number of vertices in $G\times H$, q is the number of edges in $G\times H$. We can apply this concept when we are merging the server in wireless sensor network it will give optimal solution.

Definition 1

The Cartesian product of G and H, written $G\times H$, is the graph with vertex set $V(G) \times V(H)$ specified by putting (u,v) adjacent to (u',v') if and only if i)u=u' and $vv'\in E(H)$ or ii)v=v' and $uu'\in E(H)$ [13].

Definition 2

The connected domatic number of G, denoted by $d_c(G)$, is the maximum order of a partition $\{V_1,V_2,\ldots V_k\}$ of V into connected dominating sets.

Proposition 3

If G and H is a complete graph and $G\times H$ is also a connected graph then

i) $d(G)\leq d_c(G)\leq d_c(G\times H)$

ii) $d(H)\leq d_c(H)\leq d_c(G\times H)$

Proposition 4

If G and H is a complete graph and $G\times H$ is also a connected graph then

$d_c(G\times H) = d_c(\overline{G\times H})$

Results

1. $\deg(H) \leq \deg(G) \leq \deg(G\times H)$.
2. $\deg(\overline{G\times H}) = (p_2-1)\deg(G)$, where p2 is the number of vertices in H.
3. $\deg(G\times H) = \deg(G) + \deg(H)$.
4. if $|G\times H| = p$ then $1 \leq k \leq p/2$.

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Theorem 5

For any complete graph G and H of order p_1 and p_2. Let G×H be a Cartesian product of G and H and having order $p_1 \times p_2$. Then $d_c(G\times H) = p_2$.

Theorem 6

Let G×H be a regular α-connected graph of order $p_1 \times p_2$ and degree α where $\alpha = \frac{p_1 \times p_2}{2}$ Then G (H is Hamiltonian.

Theorem 7

For any connected graph G×H, $d_c + \overline{d_c} = p$.

Proof

We know that $d_c \le \delta$.
$\overline{d_c} \le \overline{\delta} = p - 1 - \Delta$.
$\therefore d_c + \overline{d_c} = \delta + \overline{\delta} = \delta + p - 1 - \Delta$.
In Cartesian product of two graphs, should have an even number of vertices.
Since $\delta \le \overline{\delta}$, we have $r < p/2$.
Hence $\gamma_c \ge 2$. So that $d_c \le \lfloor p/2 \rfloor$.
Also $\overline{d_c} \le \lfloor p/2 \rfloor$.
And hence $d_c + \overline{d_c} \le \lfloor p/2 \rfloor + \lfloor p/2 \rfloor$
$= p$.

Theorem 8

If $\gamma_c(G\times H) \ge 2$ then $d_c(G\times H) \le \frac{p}{\gamma_c(H)}$ and the bound is sharp.

Proof

Let us assume that $d_c(G\times H) = t$ and $(D_1, D_2, \ldots D_t)$ is a partition of V(G) into t-connected dominating sets. Since each $\langle D_i \rangle$ is a connected dominating set, it follows that $|D_i| \ge \gamma_c(H)$ for i=1,2,3,…t.
Hence $p = \sum_{i \le i \le t} |D_i| \ge \gamma_c(H)$
that is, $d_c(G\times H) \le \frac{p}{\gamma_c(H)}$.

Theorem 9

For any connected graph G×H, $d_c(G\times H) \le p \times k(G\times H)$ and the bound is sharp.

Proof

Let S denote the cutest with cardinality k(G). Then <V(G)-S> is disconnected. It is obvious that any connected dominating set must contain at least p2 number of vertices of S so G (H has almost p-|S| pairwise disjoint connected dominating sets. Hence G×H, $d_c(G\times H) \le p \times k(G\times H)$.

Theorem 10

There exists a graph G for which k(G) is a circuit of an, odd length for p_2=odd number, even length for a p2= even number, then $c(G\times H) \le d_c(G\times H)$.

Proof

Let G×H be a graph with the following structure. The vertex set of G×H is $V(G\times H) = \cup_{i=1}^{p} X_i \cup Y_i$ Where $|X_i| = p-2$, $|Y_i| = 2$ for each $i \in \{1,2,3,\ldots.p\}$, $X_i \cap Y_i = \phi$ for each.
$\{i,j\} \subset \{1,2,3,\ldots p\}$, $X_i \cap X_i = Y_i \cap Y_i = \phi$ for each $\{i,j\} \subset \{1,2,3,\ldots p\}$, such that $i \neq j$. for each $i \in \{1,2,3,\ldots.p\}$the set $X_i \cup X_{i+1} \cup Y_i$ induces a clique denoted by C_i {the sum i+1 is taken modulo p}. The graph G is the union of the cliques $C_1, C_2, C_3, \ldots C_p$. Evidently the clique graph k (G (H) is a cycle on p vertices. That it may be a triangle, square, pentagon, etc., the unique element of Y_i will be denoted by Y_i. For each $i \in \{1,2,3,\ldots.p\}$. We have C (G (H) =p_1 and we shall have $dc(G\times H) \le p_1$. Hence $d_c(G\times H) \le c(G\times H)$.

Application

- Sleep-scheduling problem in wireless sensor network.
- Rotation of CDS in Ad Hoc Sensor Networks.

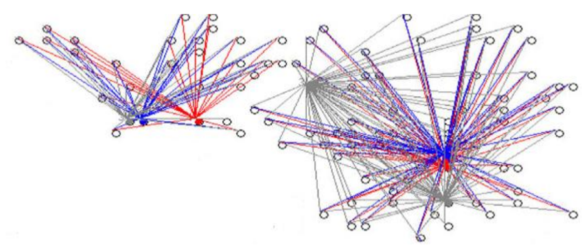

Figure 1. Three Dominating Sets in $\overline{G\times H}$, the vertices which link Black line it belongs to D_1, the vertices which link Red line it belongs to D_2, the vertices which link Blue line it belongs to D_3.

Example

Let us consider the two graphs

G

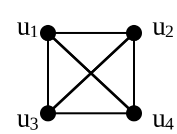

Figure 2. Complete graph on 4 vertices(K_4)

H

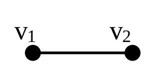

Figure 3. Complete graph on 2 vertices(K_2)

G×H

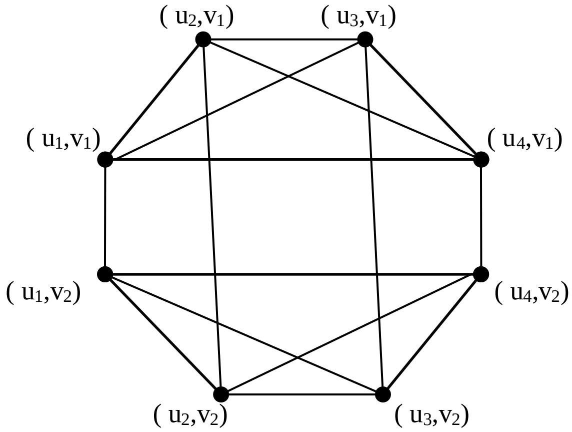

Figure 4. Cartesian Product on G and H

From the figure 3, D_1={(u_1,v_1), (u_1,v_2)}
D_2={(u_2,v_1), (u_2,v_2)}
D_3={(u_3,v_1), (u_3,v_2)}
D_4={(u_4,v_1), (u_4,v_2)}.
S={D_1,D_2,D_3,D_4}, $\gamma_c(G\times H) = 4$.

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